COMPUTERS AND MATHEMATICS: A UNIVERSAL TOOL

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1. INTRODUCTION

We shall present first an image of an archeological site obtained by the Satellite SPOT 2 and in successive images we show how informations of archeological interest can be extracted from these data by Mathematical Analysis and by the use of Computers.

This example, although simple from a mathematical viewpoint, is given as an introduction to the **general methodology** which consists in the **very close interplay** between three scientific and technological fields:

(i) data processing

(ii) mathematical modelling

(iii) extraction of informations from the mathematical models, using

(iii) 1 . mathematical analysis

(iii) 2 computers.

We shall make this interplay more precise by the way of examples.

This general methodology is the fruit of a long heritage, where some of the greatest geniuses of all times have played a decisive role.

Since the quite recent possibility offered by super-computers, by workstations, by data basis, by networks and by visualisation tools, it is becoming possible to treat by this methodology more and more evolution phenomena in 3 space dimensions. Hence a "positive feedback" appears of tremendous consequences: new problems are considered in science and in Industry from this point of view; this, in tern, implies that new methods are found, new and more powerful tools are developed; and then — the effect of positive feedback — these progresses give hope to tackle new industrial or scientific problems and so on.

We pass to examples.

2. STABILITY

This is a remark of an historical nature, with a very important lesson. During the period 1910–1922, a remarkable English meteorologist L.F. Richardson attempted (in his words, dreamed) to **compute** weather forecasts. Although a very exceptional man, his practical attempts failed. We shall explain why, and we shall also explain why the usual explanations of this relative failure are wrong. We shall show that the lesson is: the more powerful computers are, the closest we approach the "real" solution of the model provided we use **rigorous** approximate schemes. Computers "punish" instability and more generally all not well founded arguments, should it be in the data, in the modelling, in the mathematical analysis or in the software.

3. HOMOGENIZATION, NUCLEAR SAFETY, OPTIMUM DESIGN

All examples and results presented correspond to **team work** (on a National or International Basis). We shall first present composite materials with a periodic structure and explain why they call for new mathematical techniques (homogenization theory). A variant is composite geometry.

An example, coming from Nuclear Safety and provided by research workers from the EDF (Electricité de France) together with an International Team, will be shown.

Other applications of homogenization theory concern optimum design, porous media modelling, etc.

4. MULTIBODIES AND CONTROLLABILITY

"Multibodies" are here structures (such as those arising in Space or in Robotics and in many other places) which consist of several pieces (or bodies) of various sizes, dimensions, materials and which are "tied" together through junctions.

The question is: how to stabilize, to control, to regulate these structures – to make them behave as we wish! – in a way which is technically feasible, efficient and robust.

New problems arise in these directions: new mathematical techniques (asymptotics) for the modelling, new method for the control (HUM = Hilbert Uniqueness Method).

Numerical results (also joint team work) will be presented. The next challenge is: what to do in non linear cases? Can one envision the control of bifurcations, certainly a matter of importance in Industry.

5. TURBULENCE AND CLIMATOLOGY

A flow can be called "turbulent" when it shares three properties:

(i) very high sensitivity to uncertainties, in Initial or in Boundary Conditions;

(ii) increased mixing property;

(iii) a wide range of spatial wave lengths is involved.

They will be explained in non technical terms. These properties appear in "Industrial" flows — around cars, airplanes,… — or in "natural" flows — rivers, oceans, atmosphere,… —

Very interesting questions arise. One of them is: can one approximately control turbulence? This is largely an open question. A still vague conjecture is that there is some kind of "equivalence" between turbulence and chaos on one hand and approximate controllability on the other hand.

Climatology raises different but related questions: what are the attractors (if any)? The slow or the inertial varieties? What can really be computed for large time horizons (from 10 years to 1 century)?

Another crucial problem is met in climatology: the question of interfaces. Interface Atmosphere/Ocean, or Interface Liquid Ocean/Solid Ocean. This last one leads to so called free boundary problems which were studied in the recent past by using Variational Inequalities and Quasi Variational Inequalities — a tool introduced with A. BENSOUSSAN for **other** purposes (impulse stochastic control). Some phenomena of **non** controllability (i.e. of possible irreversible changes) appear here, a preoccupying observation.

6. SENTINELS AND ECOLOGY

In all phenomena concerning the Planet Earth System — should they be of a global nature or at a **local level** (such as pollution in water systems: lakes, closed seas, estuaries,…) — we have to find **signs** that "something" is happening, or not —

A new tool — called Sentinels — has been introduced for that purpose in 1988. Numerical applications will be presented.

7. FRONTIERS

In conclusion, some of the present "frontiers" will be indicated. They show **some** of the trends.